$\exp(x;\lambda) = \lambda e^{-\lambda x} = \text{Gamma}(1, \lambda)$

$\text{Gamma}(x; \alpha, \lambda) = c x^{\alpha-1} e^{-\lambda x}, \quad x > 0$

$\text{Beta}(x; \alpha, \beta) = c x^{\alpha-1} (1-x)^{\beta-1}$

$\chi_n^2 = \sum_{i=1}^{n} X_k^2 \sim N(0,1) = \text{Gamma}(\frac{n}{2}, \frac{1}{2})$

$t_n = N(0,1) / \text{sqrt}(\chi_n^2 / n)$

$\text{Cauchy} = t_1 = X/Y \qquad X, Y \sim N(0,1)$

$F_{m,n} = (\chi_m^2 / m) / (\chi_n^2 / n)$

$X \sim \text{poi}(\lambda) \Rightarrow S_n \sim \text{poi}(n\lambda)$

$X \sim N(\mu, \sigma^2) \Rightarrow S_n \sim N(n\mu, n\sigma^2)$

$X \sim \Gamma(\alpha, \lambda) \Rightarrow X \sim \Gamma(n\alpha, \lambda)$

DEFINITION **8.2** Let $\hat{\theta}$ be a point estimator for a parameter θ. Then $\hat{\theta}$ is an *unbiased estimator* if $E(\hat{\theta}) = \theta$. If $E(\hat{\theta}) \neq \theta$, $\hat{\theta}$ is said to be *biased*.

DEFINITION **8.3** The *bias* of a point estimator $\hat{\theta}$ is given by $B(\hat{\theta}) = E(\hat{\theta}) - \theta$.

DEFINITION **8.4** The *mean square error* of a point estimator $\hat{\theta}$ is

$$\text{MSE}(\hat{\theta}) = E[(\hat{\theta} - \theta)^2].$$

$$\text{MSE}(\hat{\theta}) = V(\hat{\theta}) + [B(\hat{\theta})]^2.$$

Table 8.1 Expected values and standard errors of some common point estimators

Target Parameter θ	Sample Size(s)	Point Estimator $\hat{\theta}$	$E(\hat{\theta})$	Standard Error $\sigma_{\hat{\theta}}$
μ	n	$\overline{Y}$	μ	$\frac{\sigma}{\sqrt{n}}$
p	n	$\hat{p} = \frac{Y}{n}$	p	$\sqrt{\frac{pq}{n}}$
$\mu_1 - \mu_2$	n_1 and n_2	$\overline{Y}_1 - \overline{Y}_2$	$\mu_1 - \mu_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ *†
$p_1 - p_2$	n_1 and n_2	$\hat{p}_1 - \hat{p}_2$	$p_1 - p_2$	$\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$ †

*σ_1^2 and σ_2^2 are the variances of populations 1 and 2, respectively.
†The two samples are assumed to be independent.

$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2$

estimator for σ^2

For large n, $\overline{Y}, \hat{p}, \overline{Y}_1 - \overline{Y}_2, \hat{p}_1 - \hat{p}_2 \sim N(0,1)$

Thus, the endpoints for a $100(1-\alpha)\%$ confidence interval for θ are given by

$$\hat{\theta}_L = \hat{\theta} - z_{\alpha/2}\sigma_{\hat{\theta}} \quad \text{and} \quad \hat{\theta}_U = \hat{\theta} + z_{\alpha/2}\sigma_{\hat{\theta}}.$$

$100(1-\alpha)\%$ lower bound for $\theta = \hat{\theta} - z_{\alpha}\sigma_{\hat{\theta}}$,
$100(1-\alpha)\%$ upper bound for $\theta = \hat{\theta} + z_{\alpha}\sigma_{\hat{\theta}}$.

cedures outlined in Table 8.1 is analogous to that just described. The experimenter must specify a desired bound on the error of estimation and an associated confidence level $1-\alpha$. For example, if the parameter is θ and the desired bound is B, we equate

$$z_{\alpha/2}\sigma_{\hat{\theta}} = B,$$

When σ unknown, $\sigma \approx \frac{\text{range}}{4}$

Summary of Small-Sample Confidence Intervals for Means of Normal Distributions with Unknown Variance(s)

Parameter	*Confidence Interval* ($\nu = df$)
μ	$\overline{Y} \pm t_{\alpha/2}\left(\frac{S}{\sqrt{n}}\right), \qquad \nu = n - 1.$
$\mu_1 - \mu_2$	$(\overline{Y}_1 - \overline{Y}_2) \pm t_{\alpha/2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$, where $\nu = n_1 + n_2 - 2$ and $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$ (requires that the samples are independent and the assumption that $\sigma_1^2 = \sigma_2^2$).

A 100(1 − α)% Confidence Interval for σ^2

$$\left(\frac{(n-1)S^2}{\chi^2_{\alpha/2}}, \frac{(n-1)S^2}{\chi^2_{1-(\alpha/2)}} \right)$$

for σ, take sqrt