$x^2 - 290x = 289\sqrt{x}$, compute $\sqrt{x-\sqrt{x}}$

$x^2 - x = (x+\sqrt{x})(x-\sqrt{x})$

$x^2 - x = 289(\sqrt{x}+x)$

$(x+\sqrt{x})(x-\sqrt{x}) = 289(x+\sqrt{x})$

$x - \sqrt{x} = 289$

$\sqrt{x-\sqrt{x}} = 17$

$X_1, X_2, \cdots$ i.i.d. $\mathbb{E}|X_1| < \infty \Rightarrow \lim_{n\to\infty} \bar{X}_n \overset{a.s.}{=} \mathbb{E}X_1$ (SLLN) $\leftarrow \mu$

$\mathbb{E}X_1^2 < \infty$, then $\lim_{n\to\infty} \sqrt{n}(\bar{X}_n - \mu) \overset{d}{=} N(0, \sigma^2)$ (CLT) $\leftarrow \mathrm{Var}(X_1)$

Slutsky: $X_n \xrightarrow{d} X$

$Y_n \xrightarrow{d} a = \text{constant} \neq 0 \Rightarrow \begin{cases} X_n \pm Y_n \xrightarrow{d} X \pm a \\ \frac{X_n}{Y_n} \xrightarrow{d} \frac{X}{a} \\ X_n Y_n \xrightarrow{d} Xa \end{cases}$

Ex. 1) $\sqrt{n}(\chi_n^2 - n) \xrightarrow[n\to\infty]{d} N(0,2)$

2) approx $100(1-\alpha)\%$ CI for μ when σ^2 is unknown

$\bar{X}_n \pm \frac{S_n}{\sqrt{n}} z_{\alpha/2}$ why? $\sqrt{n}(\bar{X}_n - \mu) \xrightarrow{d} N(0, \sigma^2)$

$\frac{\sqrt{n}(\bar{X}_n - \mu)}{\sigma} \xrightarrow{d} N(0,1)$

SLLN $S_n^2 \xrightarrow[n\to\infty]{a.s} \sigma^2 > 0$ slutsky $\frac{\sqrt{n}(\bar{X}_n - \mu)}{S_n} \xrightarrow{d} N(0,1)$

Def: 1) $\hat{\theta}_n$ is consistent (strongly) if $\lim_{n\to\infty} \hat{\theta}_n \overset{a.s.}{=} \theta$

2) $\hat{\theta}_n$ is asymptotically normal if $\exists\, a_n > 0, \nearrow \infty$, s.t.

$a_n(\hat{\theta}_n - \theta) \xrightarrow{d} N(0,1)$

asymp normal $\Rightarrow$ consistent

Ex. (not asymp normal)

$X \sim U(0, \theta)$

$\hat{\theta}_n = X_{(n)} < \theta$

$n(\theta - X_{(n)}) \xrightarrow{d}$ exponential

$P(n(\theta - X_{(n)}) > x) = P(\theta - X_{(n)} > \frac{x}{n}) = P(X_{(n)} < \theta - \frac{x}{n})$

$= P(\underbrace{\frac{X_{(n)}}{\theta}}_{= U_{(n)}} < 1 - \frac{x}{n\theta}) = (1 - \frac{x}{n\theta})^n \xrightarrow[n\to\infty]{} \underbrace{e^{-x/\theta}}_{= P(Y > x)}$

δ - method : if we have $a_n(\hat{\theta}_n - \theta) \xrightarrow{d} N(0,1)$
(delta)

and $f = f(x)$ $f'(\theta) \neq 0$

then $a_n(f(\hat{\theta}_n) - f(\theta)) \xrightarrow{d} N(0, |f'(\theta)|^2)$

why? Taylor $f(\hat{\theta}_n) = f(\theta) + f'(\tilde{\theta}_n)(\theta - \hat{\theta}_n)$ $\hat{\theta}_n - \theta \to 0$

$f(\hat{\theta}_n) - f(\theta) = \underbrace{f'(\tilde{\theta}_n)}_{\xrightarrow{n\to\infty} f'(\theta)} \underbrace{(\theta - \hat{\theta}_n)}_{\xrightarrow{d} N(0,1)}$ $\Leftrightarrow$ $\tilde{\theta}_n \to \theta$

$\downarrow$

$f'(\theta) N(0,1)$