$x + 2\sqrt{x} = 2$, compute $x + \frac{4}{\sqrt{x}}$

$\sqrt{x} + 2 = \frac{2}{\sqrt{x}}$

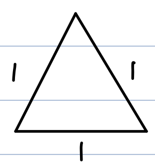
$2\sqrt{x} + 4 = \frac{4}{\sqrt{x}}$

$x + 2\sqrt{x} + 4 = x + \frac{4}{\sqrt{x}} = 6$

---

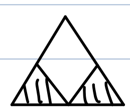
recall $\left[\sqrt{\frac{S_n^2(n-1)}{\chi^2_{n-1,\alpha/2}}}\ ,\ \sqrt{\frac{S_n^2(n-1)}{\chi^2_{n-1,1-\alpha/2}}}\right]$ → length?

larger $n \to \sim \frac{1}{\sqrt{n}}$



area $\frac{\sqrt{3}}{4} = A$

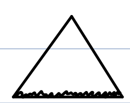
peri = 3



area = $\frac{A}{2}$

peri = 3

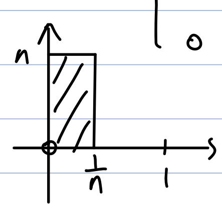
⋮



area = $\frac{A}{2^n}$

peri = 3

Ex. $f_n(x) = \begin{cases} 0 & \frac{1}{n} \le x \le 1 \\ n & 0 < x < \frac{1}{n} \\ 0 & x = 0 \end{cases}$



for fixed x, $f_n(x) \to 0$ as $n \to \infty$

but $\int_0^1 f_n(x)\,dx = 1$

$\Omega = [0,1]$

$P(a,b) = b - a$

Lebesgue measure

$X_n \xrightarrow{\boxed{M}} X$

$M = \begin{cases} \text{c.c.} \\ \text{a.s.} \\ L_p \\ p \\ d \end{cases}$

CLT

c.c. $\downarrow$ a.s. (strong $\leftarrow$ a.s.) $\rightleftarrows L_p$ ((S)LLN)

a.s. $\downarrow$ p, $L_p \to$ p

p $\downarrow$ d, d $\uparrow$ p (Sluts: if limit is nonrandom)

weak $\leftarrow$ d

$X_n \xrightarrow{d} X$ as x is continuous th $P(X_n \le x) \to P(X \le x)$, $n \to \infty$

F_X continuous $\uparrow$

if $X_n \xrightarrow{d} X$ as $Y_n \xrightarrow{d} Y$

and Y is nonrandom $P(Y=a)=1$ $a \in \mathbb{R}$

then

$X_n \pm Y_n \xrightarrow{d} X \pm Y$

$X_n Y_n \xrightarrow{d} XY$

$\frac{X_n}{Y_n} \xrightarrow{d} \frac{X}{Y}$ $(a \neq 0)$

(if Y is nonrandom, then $Y_n \xrightarrow{d} Y \Leftrightarrow Y_n \xrightarrow{P} Y$

$t_\infty \overset{d}{=} N(0,1) \xrightarrow[n\to\infty]{d} N(0,1)$

$\frac{N(0,1)}{\sqrt{\frac{\chi_n^2}{n}}}$ SLLN

$\chi_n^2 \overset{d}{=} X_1^2 + X_2^2 + \dots + X_n^2$ X_i – i.i.d. $N(0,1)$

(S)LLN $\frac{\chi_n^2}{n} \xrightarrow{a.s.} 1$

CLT: $\left(\frac{\chi_n^2}{\sqrt{n}} - \sqrt{n}\right) \xrightarrow{d} N(0,2)$

$\chi_n^2 \overset{d}{\approx} \sqrt{n}\,(N(0,2) + \sqrt{n})$

$\chi^2_{n,\alpha}$: $P(\chi_n^2 > \chi^2_{n,\alpha}) = \alpha$

$P\left(\frac{\chi_n^2}{\sqrt{n}} + n > \chi^2_{n,\alpha}\right) = \alpha$ $z_\alpha = \frac{\chi^2_{n,\alpha} - n}{\sqrt{2n}}$

$P(\sqrt{n}N(0,2) + n > \chi^2_{n,\alpha}) = \alpha$ $\chi^2_{n,\alpha} \approx n + \sqrt{2n}\, z_\alpha$