$x + \frac{1}{y} = 1, \quad \frac{1}{x} + y = 2$

compute $x^{2022} + \frac{1}{y^{2022}} = 0$

$(x + \frac{1}{y})(\frac{1}{x} + y) = 1 + 1 + xy + \frac{1}{xy} = 2$

$\frac{1}{xy} = -xy$

$x^2 = -\frac{1}{y^2}$

$x^{2(1011)} = -\frac{1}{y^{2(1011)}}$

Exact vs Asymptotic

$N(\mu, \sigma^2)$ σ - unknown

$100(1-\alpha)\%$ for μ in $\bar{X}_n \pm \frac{\sigma}{\sqrt{n}} z_{\alpha/2}$

$N(\mu, \sigma^2)$ σ - unknown

$100(1-\alpha)\%$ CI for μ $\bar{X}_n \pm \frac{S_n}{\sqrt{n}} t_{n-1, \alpha/2}$

$n \to \infty$ $S_n \approx \sigma$

$t_n = \frac{N(0,1)}{\sqrt{\frac{\chi_n^2}{n}}} \Rightarrow t_\infty = N(0,1)$

$\chi_n^2 = \sum_{k=1}^{n} z_k^2$ $z_k \sim N(0,1)$ i.i.d

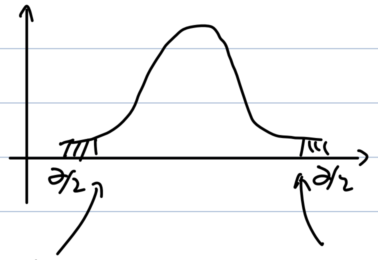
LLN: $\frac{1}{n} \sum_{k=1}^{n} z_k^2 \xrightarrow[n \to \infty]{} \mathbb{E} z_k^2 = 1$

$N(\mu, \sigma^2) \to N(1, \sigma^2)$ $\theta = \sigma$

$\theta - \sigma$ unknown $\Rightarrow \hat{\theta}_n^2 = S_n^2 = \frac{1}{n-1} \sum_{k=1}^{n} (X_k - \bar{X})^2$

μ - unknown known: $S_n^2 (n-1) \sim \sigma^2 \chi_{n-1}^2$

pivot: $\frac{S_n^2 (n-1)}{\sigma^2} \stackrel{d}{=} \chi_{n-1}^2$



$\alpha = 0.05$

$\alpha/2 = 0.025$

$\chi^2_{n-1,1-\alpha/2} \rightarrow$ small $\chi^2_{n-1,\alpha/2} \rightarrow$ large

0.975   0.025

$$P\left(\chi^2_{n-1,1-\alpha/2} < \frac{S_n(n-1)}{\sigma^2} < \chi^2_{n-1,\alpha/2}\right) = 1-\alpha$$

$$\Downarrow$$

$$P\left(\frac{S_n^2(n-1)}{\chi^2_{n-1,1-\alpha/2}} < \sigma^2 < \frac{S_n^2(n-1)}{\chi^2_{n-1,\alpha/2}}\right) = 1-\alpha$$

$100(1-\alpha)\%$ CI for σ in $\left[\sqrt{\frac{S_n^2(n-1)}{\chi^2_{n-1,\alpha/2}}}, \sqrt{\frac{S_n^2(n-1)}{\chi^2_{n-1,1-\alpha/2}}}\right]$