$x^{\frac{1}{3}}+y^{\frac{1}{3}}=z^{\frac{1}{3}}$

compute $\dfrac{xyz}{(z-x-y)^3}=\dfrac{1}{27}$

$$z=(x^{\frac13}+y^{\frac13})^3 = x+y+3x^{\frac13}y^{\frac13}(x^{\frac13}+y^{\frac13})$$
$$= x+y+3x^{\frac13}y^{\frac13}z^{\frac13}$$

$$z-x-y=3(xyz)^{\frac13}$$

$$(a+b)^3=a^3+\underbrace{3a^2b+3ab^2}_{3ab(a+b)}+b^3$$

CI v.s. point estimation

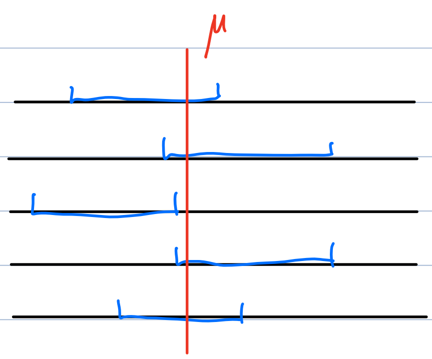
$X,\ \mathbb{E}|X|<\infty \Rightarrow \bar{X}_n=\hat{\mu}_n$

$\mu=\mathbb{E}X \qquad (\lim_{n\to\infty}\bar{X}_n=\mu)$

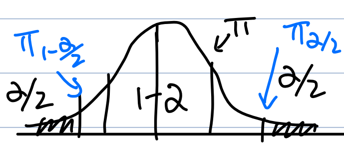
$100(1-\alpha)\%$ CI $\longrightarrow$ $\alpha=0.05$

95% CI

for 100 intervals, about 95 will cover the true value



$(X,\theta)\longrightarrow\hat{\theta}_n\longrightarrow$ pivot



$\theta\in[L_n,R_n]$

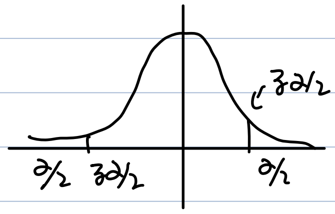
$100(1-\alpha)\%$ CI

$X \sim N(\mu, 1)$

$\hat{\mu}_n = \bar{X}_n = \frac{1}{n}\sum_{k=1}^{n} X_k \sim N(\mu, \frac{1}{n})$

pivot $\quad \bar{X}_n - \mu \sim N(0, \frac{1}{n})$

better pivot $\quad \sqrt{n}(\bar{X}_n - \mu) \sim N(0,1)$



$100(1-\alpha)\%$ CI

symmetric

$P(|\sqrt{n}(\bar{X}_n - \mu)| < z_{\alpha/2}) = 1-\alpha$

$\Updownarrow$

$P(\bar{X}_n - \frac{z_{\alpha/2}}{\sqrt{n}} < \mu < \bar{X}_n + \frac{z_{\alpha/2}}{\sqrt{n}}) = 1-\alpha$

$\Rightarrow$ CI in $[\bar{X}_n - \frac{z_{\alpha/2}}{\sqrt{n}}, \bar{X}_n + \frac{z_{\alpha/2}}{\sqrt{n}}]$ or $\bar{X}_n \pm \frac{z_{\alpha/2}}{\sqrt{n}}$

$\alpha = 0.05 \Rightarrow z_{\alpha/2} = 1.96$

$\alpha = 0.05 \Rightarrow z_{0.025} = 1.96$

$X \sim U(0, \theta)$

$100(1-\alpha)\%$ CI for θ? $\qquad \hat{\theta}_n = X_{(n)}$

$\theta > X_{(n)}$

$\Rightarrow$ CI in $[X_{(n)}, R_n] \Rightarrow R(\theta > R_n) = \alpha$

$\frac{X}{\theta} \sim U(0,1) \Rightarrow$ pivot in $\frac{X_{(n)}}{\theta}$

$P(\frac{X_{(n)}}{\theta} < t) = t^n \Leftrightarrow P(X_{(n)} < t\theta) = P(\theta > \frac{X_{(n)}}{t})$

$Y_{(1)}, \cdots, Y_{(n)}$ i.i.d. $U(0,1)$ $\qquad = \alpha \qquad t = \alpha^{1/n}$

CI in $[X_{(n)}, \frac{X_{(n)}}{\alpha^{1/n}}]$