$x+y+z=0 \Rightarrow z=-x-y$

$xyz=15$

Compute $x^3+y^3+z^3$

$x^3+y^3+z^3$

$= x^3+y^3-(x+y)^3$

$= -3x^2y-3xy^2$

$= -3xy(x+y)$

$= 3xyz = 45$

Population $X \to$ indep random variable $\boxed{\text{i.i.d. } X_1, \dots, X_n}$

Likelihood Function

Distribution of the sample

evaluated on the sample

pdf or pmf

$X \sim B(1,p) \quad x \in \{0,1\}$

pmf of x is $p^x(1-p)^{1-x}$

$L_n(\vec{x}) = \prod_{k=1}^{n} p^{x_k}(1-p)^{1-x_k}$

$\to L_n(\vec{x}) = p^{\sum_{k=1}^{n} x_k}(1-p)^{n-\sum_{k=1}^{n} x_k}$

$\downarrow$

order stats

$X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$

$\downarrow$

empirical cdf F_n

$\downarrow$

Glivenko - Cantelli

$F_n(x) \approx P(X \le x)$

$X \sim N(0,1)$

pdf $\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

pdf of $x_1 \dots x_n$

$\frac{1}{\sqrt{2\pi}} e^{-\frac{x_1^2}{2}} \cdots \frac{1}{\sqrt{2\pi}} e^{-\frac{x_n^2}{2}}$

$= \frac{1}{(2\pi)^{\frac{n}{2}}} \exp\left(-\frac{1}{2}\sum_{k=1}^{n} x_k^2\right)$

$L_n(\vec{x}) = \frac{1}{(2\pi)^{\frac{n}{2}}} \exp\left(-\frac{1}{2}\sum_{k=1}^{n} x_k^2\right)$

Estimation

an estimate $\longrightarrow$ number $\qquad \bar{X}_n = \frac{1}{5}$

estimator $\longrightarrow$ random variable $\qquad \bar{X}_n = \frac{1}{n}\sum_{k=1}^{n} X_k$

(statistic)

point $\qquad$ interval

(confidence interval) $\longrightarrow$ parameter estimation

parameter family of distributions

$N(0,1) \longrightarrow$ known

$N(0,\sigma^2) \longrightarrow$ one parameter family

$N(\mu,\sigma^2) \longrightarrow$ two parameter family

$B(1,P) \longrightarrow$ 1- para

$P(\lambda) \longrightarrow$ 1 - para

$B(n,p) \longrightarrow$ 2- para

θ — Generic parameter $\qquad \theta \in \Theta$

$\hat{\theta}_n$ — estimator (n sample)

$\text{bias}(\theta) = \mathbb{E}\,\hat{\theta}_n - \theta$

unbiased estimator — $\mathbb{E}\,\hat{\theta}_n = \theta$

quality of estimator

$MSE(\hat{\theta}_n) = \mathbb{E}(\hat{\theta}_n - \theta)^2 \neq Var(\hat{\theta}_n)$ in general

$= Var(\hat{\theta}_n)$ if unbiased

Pythagorean Thm. $MSE(\hat{\theta}_n) = Var(\theta_n) + (\text{bias}(\theta))^2$

proof. $\mathbb{E}(\hat{\theta}_n - \theta \pm \mathbb{E}\hat{\theta}_n)^2$

$= \mathbb{E}((\hat{\theta}_n - \mathbb{E}\hat{\theta}_n) + \mathbb{E}\hat{\theta}_n - \theta)^2$

$= \mathbb{E}(\hat{\theta}_n - \mathbb{E}\hat{\theta}_n)^2 + \underbrace{2\,\mathbb{E}((\hat{\theta}_n - \mathbb{E}\hat{\theta}_n)(\mathbb{E}\hat{\theta}_n - \theta))}_{0} + (\mathbb{E}\hat{\theta}_n - \theta)^2$