Gamma distribution

pdf $\quad f(x;\theta) = \underbrace{\frac{1}{\Gamma(a)\theta^{a}}}_{\text{irrelevant for MLE, N-P test}} x^{a-1} e^{-x/\theta}$

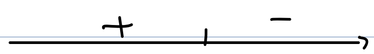
$a > 0$ known

θ unknown

$$L_n(\vec{x};\theta) = \prod_{k=1}^{n} \frac{x_k^{a-1}}{\Gamma(a)} \cdot \underbrace{\frac{1}{\theta^{na}} \exp\left(-\frac{n\bar{x}_n}{\theta}\right)}$$

$$= \exp\left(-\frac{n\bar{x}_n}{\theta} - na\ln\theta\right)$$

MLE: $\quad \dfrac{\partial \ln L_n(\vec{x};\theta)}{\partial\theta} = \dfrac{n\bar{x}_n}{\theta} - \dfrac{na}{\theta} = 0$



$$\hat{\theta}_n^{(MLE)} = \frac{\bar{x}_n}{a} \rightarrow \max$$

NP $\quad H_0: \theta = \theta_0$

$\quad\quad H_1: \theta = \theta_1 \qquad \theta_1 > \theta_0 \Rightarrow \frac{1}{\theta_1} < \frac{1}{\theta_0}$

reject H_0 if

$$\varphi = \frac{L_n(\vec{x};\theta_0)}{L_n(\vec{x};\theta_1)} \propto \exp\left(-n\bar{x}_n\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right)\right) < k$$

$n\bar{x}_n = \sum x_k \sim \text{Gamma}(na, \theta_0)$

$\varphi \rightarrow$ small $\iff n\bar{x}_n$ is large

How large? $\quad P_{\theta_0}(n\bar{x}_n > C_\alpha) = \alpha \quad \Rightarrow \quad C_\alpha = \text{Gamma}(na,\theta_0)|_\alpha$