Probability : study of uncertainty

Statistics : could be though of as "inverse probability"

→ still need some probability to study it

Beta Function : normalization of beta distribution

$$\int_0^1 x^5(1-x)^6\,dx = \frac{1}{5544} = \frac{5!\,6!}{12!} = B(6,7) = \frac{\Gamma(6)\Gamma(7)}{\Gamma(13)}$$

$$\int_0^1 x\,dx = \frac{1}{2} = \int_0^1 (1-x)\,dx$$

$$\int_0^1 x^2\,dx = \frac{1}{3} = \int_0^1 (1-x^2)\,dx$$

$$\int_0^1 x(1-x)\,dx = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\int_0^1 x(1-x)^2\,dx = \int_0^1 x^2(1-x)\,dx = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\int_0^1 \sqrt{x(1-x)}\,dx =$$

$$B(\alpha,\beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\,dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

$\alpha > 0$

$\beta > 0$

$$\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}\,dx, \qquad \Gamma(t+1) = t\,\Gamma(t)$$

$t > 0$ $\qquad \Gamma(1) = 1 \Rightarrow \Gamma(n+1) = n!$

$$\Gamma\left(\tfrac{1}{2}\right) = \int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

$$\int_0^1 \sqrt{x(1-x)}\,dx = \int_0^1 x^{\frac{1}{2}}(1-x)^{\frac{1}{2}}\,dx = B\left(\tfrac{3}{2},\tfrac{3}{2}\right) = \frac{\left(\Gamma\left(\frac{3}{2}\right)\right)^2}{\Gamma(3)} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$$