

Hypothesis testing \rightarrow "lady tasting tea"

Gaussian C.F. Gauss b. 1777 phd 1799
 $\propto e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ 1801 Ceres

Laplace $\propto e^{-\frac{|x-\mu|}{\sigma}}$

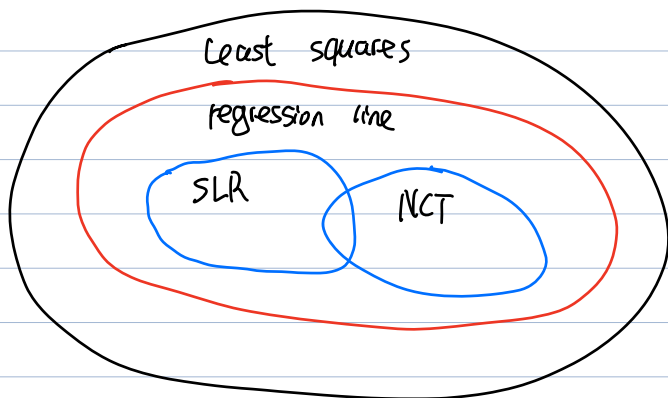
least squares $\rightarrow \sum (\text{observed} - \text{want})^2 \rightarrow \text{minimize}$

\downarrow
 $\sum |\text{observed} - \text{want}| \rightarrow \text{minimize}$

Lasso

Sparcity constraint: l_0 norm

l_p norm $(\sum |a_i|^p)^{\frac{1}{p}}$ norm $p \geq 1$
 $p=0$ # of nonzero



Three models

1) $y_i = \alpha x_i + b + \epsilon_i$ " $\epsilon_i = y_i - \alpha x_i - b$ "

Nothing is random

2) $y_i = \alpha x_i + b + \epsilon_i$ $\epsilon_i \sim \text{i.i.d } N(0, \sigma^2)$ SLR

x_i non-random

$$3) Y_i \stackrel{d}{=} \mu_Y + \rho \frac{\sigma_Y}{\sigma_X} (X_i - \mu_X) + \sigma_Y \sqrt{1 - \rho^2} \varepsilon_i \quad \varepsilon_i - \text{i.i.d. } N(0, 1)$$

$$\text{if } (X, Y) \sim N(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho) \quad \rho = \frac{E(X - \mu_X)(Y - \mu_Y)}{\sigma_X \sigma_Y}$$

can replace μ_Y with \bar{Y}

if (X, Y) is Gaussian
then $Y|X \sim N(\underbrace{\mu_Y + \rho \frac{\sigma_Y}{\sigma_X} (X - \mu_X)}_{\text{best mean-square estimator of } Y \text{ given } X}, \sigma_Y^2(1 - \rho^2))$

in particular, $E(Y|X) =$ ↓

↓
best mean-square estimator of Y given X

↓
"Gauss-Markov thm"