Likelihood ratio test (1 side alternative)

N-P test

"our" example: $X \sim Gamma(a, \frac{1}{\theta})$

a- known, $\mathbb{E}X = a\theta$

$H_0: \theta = \theta_0$

$H_a: \theta = \theta_a$

$\varphi_n = \dfrac{L_n(\vec{x}; \theta_0)}{L_n(\vec{x}; \theta_a)}$, reject H_0 when φ_n is small

if $\theta_0 > \theta_a$, then reject H_0 if $\bar{X}_n$ is small

if $\theta_0 < \theta_a$, then reject H_0 if $\bar{X}_n$ is large

"the same" in "all other examples"

1) $X \sim N(\theta, 1)$
2) $X \sim B(1, \theta)$
3) $X \sim P(\theta)$

— all reduced to $\bar{X}_n$

4) $X \sim N(0, \theta)$ — use $\sum_{k=1}^{n} X_k^2$

NP test depends on H_0: to set RR given α

can also depends on H_a

If not, then — UMP

all our example

More sophisticated alternative: Likelihood ratio test

Our setting:

$H_0: \theta \in \Omega_0$ $\qquad \Omega_0 \cap \Omega_1 = \phi$

$H_1: \theta \in \Omega_1$ $\qquad \Omega_0 \cup \Omega_1 =$ "everything"

LR : $\lambda_n = \dfrac{\sup_{\theta \in \Omega_0} L_n(\vec{x}_n; \theta)}{\sup_{\theta \in \Omega_0 \cup \Omega_1} L_n(\vec{x}_n; \theta)}$

LR test: reject H_0 if λ_n is small

RR: $\sup_{\theta \in \Omega_0} P_{\theta_0}(\lambda_n < C_\alpha) \overset{(\leq)}{=} \alpha$

Fact: $\lambda_n \leq 1$ always

Main assumption:

if $H_0: \theta = \theta_0$, then "top" of λ_n is $L_n(\vec{x}; \theta_0)$

and then $H_a: \theta \neq \theta_0$ and "bottom" of λ_n is $L_n(\vec{x}_n; \hat{\theta}_n^{(MLE)})$

$H_0: \theta \geq \theta_0$

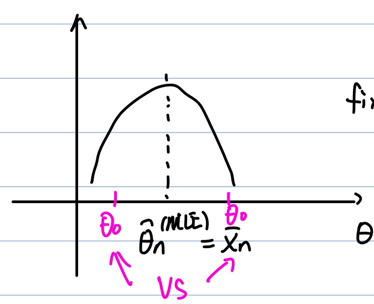
$H_a: \theta < \theta_0$ then $\lambda_n = 1$ sometimes

when $\lambda_n < 1$, then use $\hat{\theta}_n^{(MLE)}$

$X \sim \exp(\text{mean } \theta)$

$H_0: \theta = \theta_0 \iff \theta \leq \theta_0$

$H_a: \theta > \theta_0$



fixed $\vec{x}_n$

$$\lambda_n = \frac{\max_{\theta \leq \theta_0} L_n(\vec{x}; \theta)}{\underbrace{\max_{\theta > 0} L_n(\vec{x}; \theta)}_{= L_n(\bar{x}_n; \hat{\theta}_n^{(MLE)})}}$$

where the numerator $= \begin{cases} L_n(\bar{x}_n; \theta_0) & \theta_0 < \bar{x}_n \\ L_n(\vec{x}_n; \bar{x}_n) & \theta_0 > \bar{x}_n \end{cases}$

$$\lambda_n = \begin{cases} 1 & \theta_0 > \bar{x}_n \\ \dfrac{L_n(\vec{x}_n; \theta_0)}{L_n(\vec{x}_n; \bar{x}_n)} & \theta_0 < \bar{x}_n \end{cases}$$