$$x^{-x^{1-x}} = 256 = 2^{2^3}$$

$$x = -2$$

N-P test

$H_0 : \theta = \theta_0$

$H_1 : \theta = \theta_1$

when $X \sim \exp(\text{mean } \theta)$

$\theta_0 > \theta_1$

$\bar{X}_n \approx \theta_1 \Rightarrow$ reject H_0 if $\bar{X}_n$ is small

$$L_n(\vec{x};\theta) \propto e^{-\frac{1}{\theta}\sum_{k=1}^{n} X_k} = e^{-\frac{n}{\theta}\bar{X}_n}$$

$$\frac{L_n(\vec{x};\theta_0)}{L_n(\vec{x};\theta_1)} \propto \underbrace{\exp\left(-n\bar{X}_n\left(\frac{1}{\theta_0}-\frac{1}{\theta_1}\right)\right)}_{}$$

reject H_0 if $\downarrow$ small

$\theta_0 > \theta_1 \Rightarrow \frac{1}{\theta_0} < \frac{1}{\theta_1} \Rightarrow \frac{1}{\theta_0} - \frac{1}{\theta_1} < 0 \Rightarrow -\left(\frac{1}{\theta_0} - \frac{1}{\theta_1}\right) > 0$

$\exp(\text{positive } \bar{X}_n) \to$ small means $\bar{X}_n \to$ small

$\Rightarrow$ N-P test : reject H_0 if $\bar{X}_n < C_\alpha$

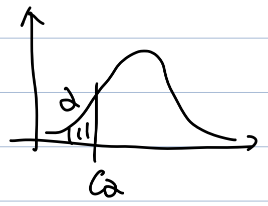
how to compute C_α ?

$P_{\theta_0}(\bar{X}_n < C_\alpha) = \alpha$

1) under θ_0, $X \sim \exp(\text{mean } \theta_0) = \text{gamma}(1, \frac{1}{\theta_0})$

$$n\bar{X}_n = \sum_{k=1}^{n} X_k \sim \text{Gamma}(n, \tfrac{1}{\theta_0})$$

$\Rightarrow C_\alpha = \text{Gamma}(n, \frac{1}{\theta_0})\big|_{1-\alpha} \to$ table computes



2) Note pdf of $n\bar{X}_n$ $\text{gamma}(n, \frac{1}{\theta_0})$

$\propto x^{n-1} e^{-x/\theta_0}$

$\Rightarrow$ pdf of $\frac{n\bar{X}_n}{\theta_0} \propto x^{n-1} e^{-x}$ $\quad \text{gamma}(n, 1)$

$\Rightarrow$ reject H_0 if $\frac{n\bar{X}_n}{\theta_0} < \tilde{c}_\alpha$

$\tilde{c}_\alpha = \text{gamma}(n,1)\big|_{1-\alpha}$

3) $\chi^2_n = \text{gamma}(\frac{n}{2}, \frac{1}{2})$

$\Downarrow$

$\frac{2n\bar{X}_n}{\theta_0} = \chi^2_{2n}$ $\Rightarrow$ reject H_0 if $\frac{2n\bar{X}_n}{\theta_0} < \chi^2_{2n, 1-\alpha}$

4) Approx. $\quad \bar{X}_n \approx N(\theta_0, \frac{\theta_0}{n})$

reject H_0 if $\bar{X}_n < \bar{C}_\alpha \Leftrightarrow \frac{\sqrt{n}\,\bar{X}_n - \theta_0}{\sqrt{\theta_0}} < \frac{\sqrt{n}\,\bar{C}_\alpha - \theta_0}{\sqrt{\theta_0}}$

$\downarrow \approx N(0,1)$ $\qquad$ $= -z_\alpha$