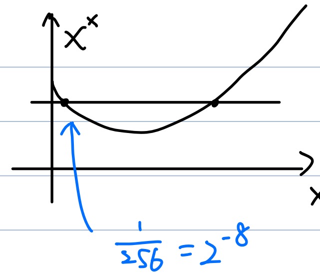
$x^x = \frac{1}{\sqrt[32]{2}}$

$= 2^{\frac{1}{2^5}} = 2^{2^{-5}}$

$= (2^{-8})^{2^{-8}}$



HT (hypothesis testing)

1. H_0, H_1

$H_0 \to$ type I, $H_1 \to$ type II

$1-\beta$ power (in general depends on particular H_1)

type I $\to$ fix $\leftrightarrow$ size/level of significance of the test

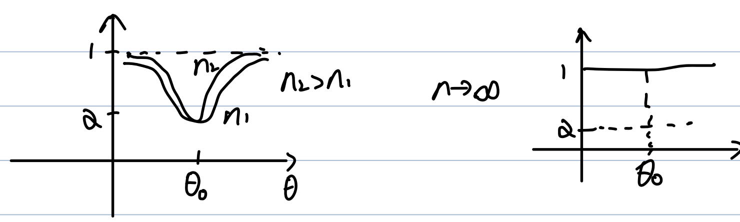
$\max_{\alpha\text{-fixed}} (1-\beta) \to$ UMP (uniformly most powerful test)

α depends only on H_0

$H_0: \theta = \theta_0$

$H_1: \theta \neq \theta_0$

$(\theta > \theta_0 \quad \theta < \theta_0)$



$\alpha = 0 \Rightarrow \beta = 1$

$\beta = 0 \Rightarrow \alpha = 1$

$\Rightarrow$ $\left.\begin{array}{l} H_0 \text{ is true} \\ H_1 \text{ is true} \end{array}\right\}$ are not random events

$\left.\begin{array}{l} P(A|H_0) \\ P(B|H_1) \end{array}\right\}$ are not conditional probability

$P(A \text{ with } \theta = \theta_0) \to \alpha$

$P(B \text{ with } \theta \neq \theta_0, \text{fixed}) \to \beta$

Pivot v.s. Test Statistics

Pivot	Test Statistics
↓	↓ $H_0: \theta = \theta_0$
depend on θ	$\varphi(\theta_0)$
↓	↓ $\swarrow J_\alpha$
"manipulate" it	rejection region ⟷ "extreme" values of $\varphi(\theta_0)$
get $100(1-\alpha)\%$ CI I_α for θ	$\alpha = P(\varphi(\theta_0) \in J_\alpha)$
↓	↓
reject H_0 if $\theta_0 \in I_\alpha$ at level α	observed value of φ: φ_0
↓	↓
p value?	p-value = $P(\varphi(\theta_0)$ is more extreme than $\varphi_0)^3$ (I refer $P(\varphi(\theta_0) > \varphi_0)$)

significant : P-value ≤ 0.05

high significance : P-value ≤ 0.01

Ex. $X \sim B(1, p)$

$H_0: p = \frac{1}{2}$

$H_1: p > \frac{1}{2}$

total # guesses

$$\varphi = \frac{-\frac{n}{2} + n_c}{\sqrt{n \cdot \frac{1}{2} \cdot \frac{1}{2}}} \approx N(0,1) \quad \text{CLT}$$

← # correct guesses

p_0 $1-p_0$

Var

Ex. $n = 100$, $n_c = 55$, $\varphi = \frac{55-50}{\sqrt{\frac{100}{4}}} = 1$

p-value : $P(N(0,1) > 1) \approx 0.16$