consistency: unbiased + Var $\to 0$

$\Omega = \{w\}$

$P(\lim_n X_n(w) = x(w)) = 1$ almost sure convergence

MVUE: minimum variance unbiased estimator, $\hat{\theta}$ unbiased

$\mathbb{E}[\hat{\theta}|u]$, u - suff stat

$X_1, \cdots, X_n$ $\hat{\mu} = \bar{x} = \frac{1}{n}\Sigma x_i$

MM: method of moments

MLE: maximum likelihood estimator

9.19 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\theta > 0$. Show that $\overline{Y}$ is a consistent estimator of $\theta/(\theta+1)$.

$\mathbb{E}\bar{Y} = \mathbb{E}Y_1 = \int_0^1 \theta y y^{\theta-1}\,dy = \frac{\theta}{\theta+1}$ unbiased

$Var(\bar{y}) = \frac{1}{n} Var(Y_1) = \frac{1}{n}(\mathbb{E}Y_1^2 - (\mathbb{E}Y_1)^2) \to 0$ as $n \to \infty$ Var $\to 0$

by theorem 9.1, $\bar{Y}$ is consistent