large / small sample?

$\downarrow$ CLT $\qquad$ $\downarrow$ assume normal

$$\frac{\hat{\theta} - \mathbb{E}\hat{\theta}}{\sigma_{\hat{\theta}}} \sim N(0,1) \qquad \frac{\hat{\theta} - \mathbb{E}\hat{\theta}}{S} \sim t_{n-1} \quad (n \geq 30, \approx N(0,1))$$

8.82 Scholastic Assessment Test (SAT) scores, which have fallen slowly since the inception of the test, have now begun to rise. Originally, a score of 500 was intended to be average. The mean scores for 2005 were approximately 508 for the verbal test and 520 for the mathematics test. A random sample of the test scores of 20 seniors from a large urban high school produced the means and standard deviations listed in the accompanying table:

	Verbal	Mathematics
Sample mean	505	495
Sample standard deviation	57	69

a Find a 90% confidence interval for the mean verbal SAT scores for high school seniors from the urban high school.

b Does the interval that you found in part (a) include the value 508, the true mean verbal SAT score for 2005? What can you conclude?

c Construct a 90% confidence interval for the mean mathematics SAT score for the urban high school seniors. Does the interval include 520, the true mean mathematics score for 2005? What can you conclude?

a) $\hat{\mu}_v = \bar{X} \pm t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} = 505 \pm (1.729)\left(\frac{57}{\sqrt{20}}\right)$

$$\frac{\bar{X} - \mu_v}{\sigma/\sqrt{n}} \sim N(0,1) \qquad \frac{\bar{X} - \mu_v}{S/\sqrt{n}} \sim t_{n-1}$$

Exp. Let $Y \sim f(y) = \begin{cases} \frac{2(\theta - y)}{\theta^2}, & 0 < y < \theta \\ 0, & \text{else} \end{cases}$

a) Show that Y/θ is a pivotal quantity.
b) Use quantity from part a) to find 90% conf. interval for θ.

$$P\left(a < \frac{Y}{\theta} < b\right) = 0.9$$

$$F_Y(y) = \int_0^y \frac{2(\theta - y)}{\theta^2}\, dy = \frac{y(2\theta - y)}{\theta^2} \qquad y \in (0, \theta)$$

$$P\left(\frac{Y}{\theta} < b\right) = P(Y < b\theta) = \frac{b\theta(2\theta - b\theta)}{\theta^2} = 2b - b^2 = 0.95$$

$$P\left(\frac{Y}{\theta} < a\right) = 2a - a^2 = 0.05$$

$$\theta \in \left(\frac{Y}{b}, \frac{Y}{a}\right)$$