8.39 Suppose that the random variable Y has a gamma distribution with parameters $\alpha = 2$ and an unknown β. In Exercise 6.46, you used the method of moment-generating functions to prove a general result implying that $2Y/\beta$ has a χ^2 distribution with 4 degrees of freedom (df). Using $2Y/\beta$ as a pivotal quantity, derive a 90% confidence interval for β.

$\frac{2Y}{\beta} \sim \chi^2_4$

$P(a < \frac{2Y}{\beta} < b) = 0.9 = 1-2$

$a = 0.71, \quad b = 9.49$

$\frac{a}{2Y} < \frac{1}{\beta} < \frac{b}{2Y}$

$\frac{2Y}{9.49} < \beta < \frac{2Y}{0.71}$ 90% CI

8.56 Is America's romance with movies on the wane? In a Gallup Poll[5] of $n = 800$ randomly chosen adults, 45% indicated that movies were getting better whereas 43% indicated that movies were getting worse.

a Find a 98% confidence interval for p, the overall proportion of adults who say that movies are getting better.

b Does the interval include the value $p = .50$? Do you think that a majority of adults say that movies are getting better?

a. $\hat{p} = \frac{x}{n}$ $\quad x \sim Bin(n,p)$

$E\hat{p} = p$, $Var(\hat{p}) = \frac{1}{n^2} np(1-p) = \frac{p(1-p)}{n}$

CLT, $\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \sim N(0,1)$

$\alpha = .02$, $P(-z_{\alpha/2} < \frac{\hat{p}-0.45}{\sqrt{\frac{(0.45)(0.55)}{800}}} < z_{\alpha/2}) = 98$

CI: 0.45 ± 0.041

8.74 Suppose that you want to estimate the mean pH of rainfalls in an area that suffers from heavy pollution due to the discharge of smoke from a power plant. Assume that σ is in the neighborhood of .5 pH and that you want your estimate to lie within .1 of μ with probability near .95. Approximately how many rainfalls must be included in your sample (one pH reading per rainfall)? Would it be valid to select all of your water specimens from a single rainfall? Explain.

$0.1 = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ $\quad\quad$ $\alpha = .05$

$0.1 = (1.96) \frac{0.5}{\sqrt{n}}$

$n = \frac{1.96^2 \cdot 0.5^2}{0.1^2} = 97$