SLR : $y = \beta_0 + \beta_1 x + \varepsilon$

$\mathbb{E} y = \beta_0 + \beta_1 x$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$

11.3 Fit a straight line to the five data points in the accompanying table. Give the estimates of β_0 and β_1. Plot the points and sketch the fitted line as a check on the calculations.

y	3.0	2.0	1.0	1.0	0.5
x	-2.0	-1.0	0.0	1.0	2.0

$\bar{x} = 0$, $\bar{y} = 1.5$, $S_{xx} = 10$, $S_{xy} = -6$

$\hat{y} = 1.5 - 0.6x$

11.20 Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent normal random variables with $E(Y_i) = \beta_0 + \beta_1 x_i$ and $V(Y_i) = \sigma^2$, for $i = 1, 2, \ldots, n$. Show that the maximum-likelihood estimators (MLEs) of β_0 and β_1 are the same as the least-squares estimators of Section 11.3.

$$L(\beta_0, \beta_1) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \exp(-(y_i - (\beta_0 + \beta_1 x_i))^2 / 2\sigma^2)$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^n e^{-\frac{1}{2\sigma^2}\sum (y_i - (\beta_0 + \beta_1 x_i))^2}$$

maximize $L(\beta_0, \beta_1)$ $\Leftrightarrow$ minimize $\sum_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))^2$

11.10 Suppose that we have postulated the model

$$Y_i = \beta_1 x_i + \varepsilon_i \qquad i = 1, 2, \ldots, n,$$

where the ε_i's are independent and identically distributed random variables with $E(\varepsilon_i) = 0$. Then $\hat{y}_i = \hat{\beta}_1 x_i$ is the predicted value of y when $x = x_i$ and $\text{SSE} = \sum_{i=1}^{n} [y_i - \hat{\beta}_1 x_i]^2$. Find the least-squares estimator of β_1. (Notice that the equation $y = \beta x$ describes a straight line passing through the origin. The model just described often is called the *no-intercept* model.)

normal equation $X^T y = X^T X \hat{\beta}$ $\Rightarrow$ $\hat{\beta} = (X^T X)^{-1} X^T y$ if invertible