Review:

x_{t+1}

x_t, u_t

$\boxed{\text{No uncertainty}}$

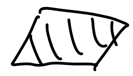
$d_t \rightarrow$ distribution

x_{t+1}

x_t, u_t

$\boxed{\text{probabilistic}}$

$d_t \in \mathcal{E}_t \leftarrow$ set



x_{t+1}

x_t, u_t

$\boxed{\text{non-deterministic}}$

Type of Uncertainty

unstructured uncertainty

→ uncertainty is not characterized / it is not modeled in an informed way

→ often, an additive model of uncertainty is used

$$x_{t+1} = f_D(x_t, u_t) + d_t$$

(parametric)

structured uncertainty

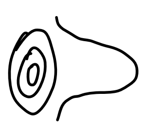
→ uncertainty enters the dynamics in an "informed" way, often as uncertain parameter

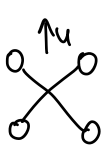
→ "functional form" is typically known

Ex. unstructured uncertainty in the longitudinal quadrotor motion

probabilistic

$d_t \sim N(0, \sigma^2)$





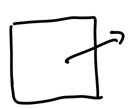
p_z, v_z

non-deterministic

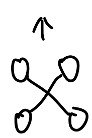
$d_t \in [-2, 2]$

$\dot{p}_z = v_z$

$\dot{v}_z = g + k_T u + k_0$



→ x_{t+1} belongs to "forward reachable set"



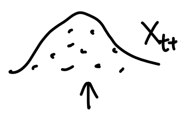
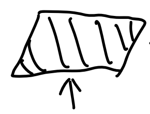
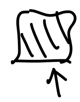
x_t, u_t

Ex. structured uncertainty

$$\dot{V}_z = g + k_T u + k_0$$

uncertainty in the value of k_T can be modeled as a Gaussian set, etc.

Uncertainty Matrix

	probabilistic	non-deterministic
unstructured	$x_{t+1} = f(x_t, u_t) + d_t$  x_{t+1} x_t, u_t	$x_{t+1} = f(x_t, u_t) + d_t$  x_{t+1} x_t, u_t
structured	$x_{t+1} = f(x_t, u_t, d_t)$  x_{t+1} x_t, u_t	$x_{t+1} = f(x_t, u_t, d_t)$  x_{t+1} x_t, u_t

probabilistic

* probabilistic safety assurances
* optimise expected cost/ reward/ performance
* probability bounds on failure

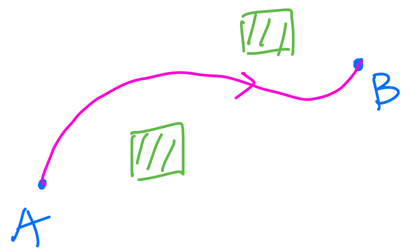
non-deterministic

* robust as "worst-case" guarantees
* optimise worst-case cost/performance
* failure impossibility conditions

Optimal Control Problem

* problem of optimizing decision-making with respect to some performance criterium

Ex.

1. 

Go from A to B as quickly as possible without collision.

2. Finding minimum energy walking gait for a bipedal robot.

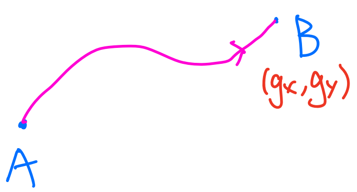
3. An autonomous helicopter to clone an acrobatic maneuver.

Minimizing some cost function

subject to
- system dynamics
- other constraints
  - control bounds
  - obstacles

} optimization problem

Ex. Go from A to B with minimum fuel.



cost function: $\overbrace{\left\| \begin{pmatrix} p_x^T \\ p_y^T \end{pmatrix} - \begin{pmatrix} g_x \\ g_y \end{pmatrix} \right\|}^{\text{terminal cost}} + \overbrace{\lambda \sum_{t=0}^{T} |u_t|^2}^{\text{running cost}}$

subject to:

$$\left.\begin{aligned} \dot{p}_x &= v\cos\theta \\ \dot{p}_y &= v\sin\theta \\ \dot{\theta} &= u \end{aligned}\right\} \text{dynamics}$$

$$|u| \le a \quad \} \text{ constraint}$$

Discrete-time OC problem

running cost

$$\min_{u_{t:T}} J(x_t, u_{t:T}, t) = \sum_{s=t}^{T-1} L(x_s, u_s) + \ell(x_T)$$

← terminal cost

$$\text{subject to} \quad \left.\begin{array}{l} x_{s+1} = f_D(x_s, u_s) \\ \underline{u} < u_s < \bar{u} \end{array}\right\} \forall s \in \{t, t+1, \dots, T$$

Here, J is the total cost accumulated over the horizon $\{t, t+1, \dots, T\}$ starting from state x_t at time t, and apply control $\{u_t, u_{t+1}, \cdots, u_T\}$

Reinforcement learning and OC try to solve the same problem, but under different assumptions in various ways.

Continuous

$$\min_{u(\cdot)} J(x(t), u(t), t) = \int_t^T L(x(s), u(s))\, ds + \ell(x(T))$$

$s=t$

$$\text{subject to} \quad \left.\begin{array}{l} \dot{x}(s) = f_c(x(s), u(s)) \\ \underline{u} \le u(s) \le \bar{u} \end{array}\right\} \forall s \in (t, T]$$

Methods

1. Calculus of Variation
2. $\boxed{\text{Dynamic Programming}}$
3. Model predictive control (receding horizon)
4. Reinforcement Learning