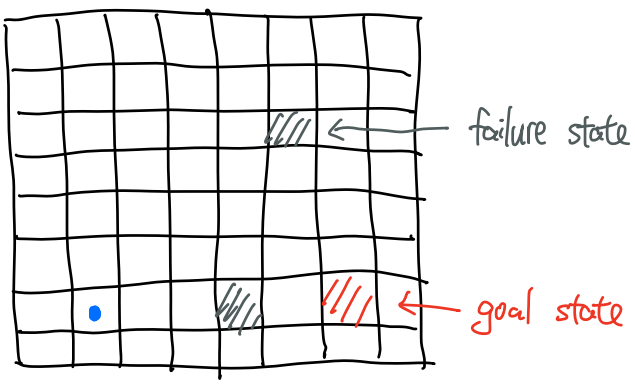
Review

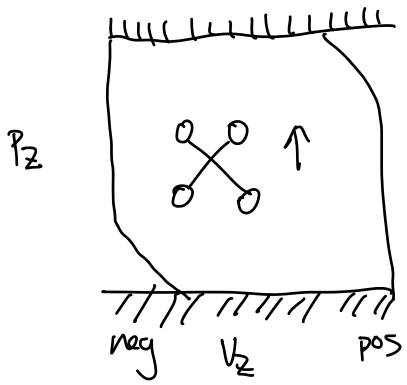
	continuous	Discrete
state	$x(t) \in \mathbb{R}^n$	x_t
control	$u(t) \in \mathbb{R}^m$	u_t
dynamics	$\dot{x}(t) = f_c(x(t), u(t))$ $| $ $\frac{dx}{dt}$	$x_{t+1} = f_D(x_t, u_t)$
trajectory	$x^u_{x_0,t_0}(t)$	$x^u_{x_0,x_0}(t)$ ← discrete timestep

Grid world



Reasons for Safety

#1 inevitable collision



∃ states from which collision is inevitable despite your best control

uncertainty

Even though we have a mathematical model of the system, it will never capture the behavior of the actual system.

our simple model

$$\dot{p}_z = v_z$$

$$\dot{v}_z = g + k_T u + k_\Phi$$

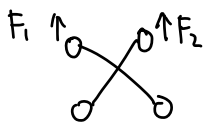
advanced model

$$\dot{p}_z = v_z$$

$$\dot{v}_z = g + k \cos\Phi(F_1, F_2)$$

$$\dot{\Phi} = w$$

$$\dot{w} = k(F_1 - F_2)$$



This is still not sufficient!

sophistication of model → ← tractability of analysis

(model) uncertainty representation

probabilistic

uncertainty is modeled as a distribution

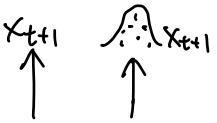
* In discrete time, we have

$$x_{t+1} = f(x_t, u_t, d_t)$$

r.v. ↑ disturbance

x_{t+1} is a random variable where

$$x_{t+1} \sim P(x_{t+1} \mid x_t, u_t)$$



non-deterministic

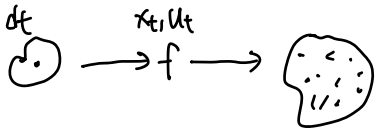
uncertainty belongs to a set

$d_t \in \mathcal{E}_t$ ← set in $\mathbb{R}^m$

* In discrete time, we have

$$x_{t+1} = f(x_t, u_t, d_t)$$

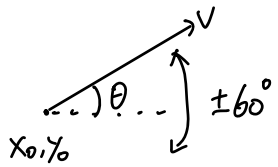
↑ some set (not a point)



x_t, u_t x_t, u_t

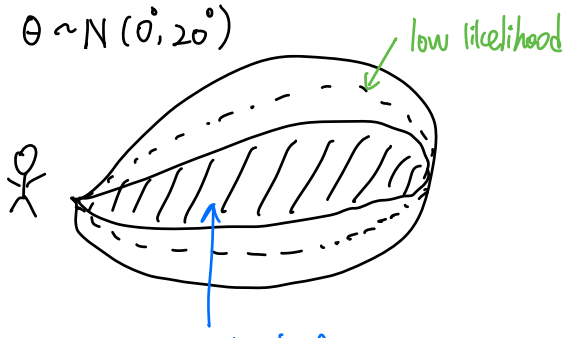
Example: human-robot interaction

$$\begin{bmatrix} \dot{p}_x \\ \dot{p}_y \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \end{bmatrix}$$



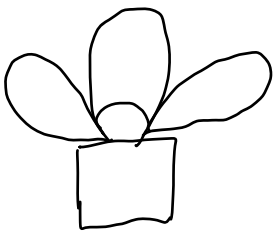
probabilistic

$\theta \sim N(0°, 20°)$

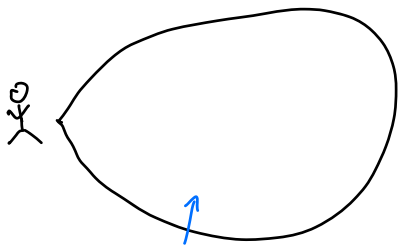


high likelihood future states of human

1. distribution of uncertainty might not be correct
2. distribution propagation is challenging, especially when the distribution becomes multi-model → mixture models



non-deterministic



all possible future states of human

"forward reachable set"

1. sets can quickly grow in size resulting in very conservative plans
   ↑ closed-loop policies are important
2. no belief update over uncertainty
3. set propagation is very challenging
   ↳ level-set methods