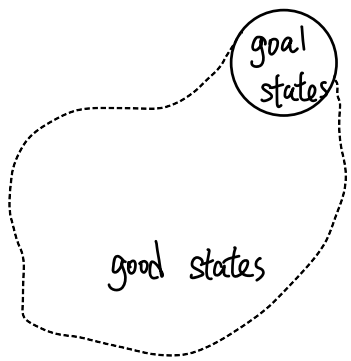
The Liveness Problem

A controller that constantly makes progress towards goal states despite the worst-case uncertainty.



want to reach goal states

$$BRT(t) = \{ x : \forall d(\cdot), \exists u(\cdot), \mathcal{Z}^{u,d}_{x,t}(s) \in \mathcal{L} \text{ for some } s \in [t,T] \}$$

$$\ell(x) \to x \in \mathcal{L} \Leftrightarrow \ell(x) \le 0$$

$$J(x,t,u(\cdot),d(\cdot)) = \min_{s \in [t,T]} \ell(x(s))$$

$$V(x,t) = \min_{u(\cdot)} \max_{d(\cdot)} J(x,t,u(\cdot),d(\cdot))$$

$$\min\left\{ \frac{\partial V}{\partial t} + \min_{u} \max_{d} \frac{\partial V}{\partial x} \cdot f(x,u,d),\ \ell(x) - V(x,t) \right\} = 0$$

$$V(x,T) = \ell(x)$$

$$u^*_{live}(x) = \operatorname*{argmin}_{u} \max_{d} \frac{\partial V}{\partial x} \cdot f(x,u,d)$$